Math 72 5.1 Polynomial Functions
& 5.2 Multiplication of Polynomials

Objectives

5.1
1) Add polynomials by combining like terms
2) Subtract polynomials by
   - distribute negative to subtract
   - combine like terms
3) Evaluate a polynomial function.

5.2
4) Multiply
   - a monomial by a polynomial using distribute
   - a binomial by a binomial using FOIL, boxes, or "double distribute" or arrows
   - a polynomial by a polynomial using boxes, arrows, or multiple distributes.
5) Notice that some special products have recognizable patterns:

$$(a+b)(a-b)$$
$$(a+b)(a+b)$$
$$(a-b)(a-b)$$
$$(a-b)(a^2+ab+b^2)$$
$$(a+b)(a^2-ab+b^2)$$

1. Add $(-3x^2+2x-7)+(4x^2-x+1)$

$= -3x^2+4x^2+2x-x-7+1$ combine like terms

$= \boxed{x^2+x-6}$

★ Like Terms ★

- added or subtracted
- same variables
- same exponents on those variables.
- variables & exponents do not change after being combined.

2. Find the opposite of $4x^2-x+1$

$-(4x^2-x+1)$

$= \boxed{-4x^2+x-1}$

- all terms must change sign
- this is often called "distribute the negative"

3. Subtract $(-3x^2+2x-7)-(4x^2-x+1)$

$= -3x^2+2x-7-4x^2+x-1$ dist neg

$= \boxed{-7x^2+3x-8}$ combine like terms.

Polynomial functions $P(x) =$ sum of terms having

- exponents which are positive
- exponents which are whole numbers
  $x^0=1$ is ok!
- coefficients can be any real #s.

ex. $p(x)=5x^3-x+10$

$p(x)=1-2x$

$p(x)=6$

not a polynomial:

$f(x)=x^{-2.5}+1$

$f(x)=\frac{3}{x-1}$

④ Evaluate $f(x) = 4x - x^3$ when $x = -2$

$$f(-2) = 4(-2) - (-2)^3$$
$$= -8 - (-8)$$
$$= -8 + 8$$
$$= \boxed{0}$$

⑤ Given $f(x) = 3x^2 - 1$ and $g(x) = -x^2 - 5$

a) find $f(x) + g(x)$ also called $(f+g)(x)$

b) find $f(x) - g(x)$ also called $(f-g)(x)$

a) $f(x) + g(x)$

$= (3x^2 - 1) + (-x^2 - 5)$ $\quad$ $+(-x^2)$ is $-x^2$

$= 3x^2 - 1 - x^2 - 5$ $\quad$ combine like terms

$= \boxed{2x^2 - 6}$

b) $f(x) - g(x)$

$= (3x^2 - 1) - (-x^2 - 5)$ $\quad$ substitute using parentheses

$= 3x^2 - 1 + x^2 + 5$ $\quad$ dist neg

$= \boxed{4x^2 + 4}$

Simplify

⑥ $3(10 + 2x)$

$= 3 \cdot 10 + 3(2x)$ $\quad$ distribute 3

$= \boxed{30 + 6x}$

or $\boxed{6x + 30}$

(7) $-2(x-5y)$

$= -2\cdot x - (-2)(5y)$ distribute -2.

$= \boxed{-2x+10y}$

OR $\boxed{10y-2x}$

(8) $(5x-1)(4)$

$= 5x\cdot 4 - 1\cdot 4$ distribute 4

$= \boxed{20x-4}$

(9) $-3x^2 \cdot 5x^6$

$-3x^2$ is a <u>monomial</u> (one term)

$= (-3)\cdot x^2 \cdot (5)\cdot x^6$

$= (-3)\cdot(5)\cdot x^2\cdot x^6$

commutative property of multiplication

$x^2\cdot 5 = 5x^2$

order doesn't matter when multiplying.

$= -15x^{2+6}$

$= \boxed{-15x^8}$

exponent law

$x^n\cdot x^m = x^{n+m}$

(10) $(4x^4y)(2x^2y^5)$

$= 4\cdot 2\cdot x^4\cdot x^2\cdot y\cdot y^5$ multiply like with like

$= 8\cdot x^{4+2}\cdot y^{1+5}$ exponent law

$= \boxed{8x^6y^6}$

(11) $3y^3(y-2y^3)$

$= 3y^3\cdot y - 3y^3\cdot 2y^3$ distribute $3y^3$

$= \boxed{3y^4 - 6y^6}$

multiply like with like

use exponent law

(12) $-mn(4m^5 - n)$

$= -mn \cdot 4m^5 - mn(-n)$ dist $-mn$

$= \boxed{-4m^6n + mn^2}$

(13) $(x^3)^2$

$= x^{3 \cdot 2}$

$= \boxed{x^6}$

exponent law
$(x^n)^m = x^{n \cdot m}$

(14) $(3x)^2$

$= 3^2 \cdot x^2$

$= \boxed{9x^2}$

exponent law
$(ab)^n = a^n \cdot b^n$

(15) $(2x^2)^3$

$= 2^3 \cdot (x^2)^3$

$= 8 \cdot x^{2 \cdot 3}$

$= \boxed{8x^6}$

(16) $(-m^2n)^4$

$= (-1)^4 \cdot (m^2)^4 \cdot n^4$

$= +1 \cdot m^{2 \cdot 4} \cdot n^4$

$= \boxed{m^8n^4}$

(17) $(x+2)(x+3)$ dist x and dist 2

$= x(x+3) + 2(x+3)$

$= x^2 + 3x + 2x + 6$ combine like terms

$= \boxed{x^2 + 5x + 6}$

(18) $(x-1)(2x-2)$

option 2:
Acronym F.O.I.L.
F= first
O= outer or outside
I= inner or inside
L= last

$= x\cdot 2x + x(-2) + (-1)(2x) + (-1)(-2)$
F O I L

$= 2x^2 - 2x - 2x + 2$

$= \boxed{2x^2 - 4x + 2}$ combine like terms

* CAUTION: F.O.I.L. only works when multiplying 2 terms by 2 terms

(19) $(1-6x)(2-x)$

$= 1\cdot 2 + 1\cdot(-x) + (-6x)\cdot 2 + (-6x)(-x)$

$= 2 - x - 12x + 6x^2$

$= \boxed{2 - 13x + 6x^2}$

OR $\boxed{6x^2 - 13x + 2}$

(20) $(3x^2+2)(x-2)$

$= 3x^2\cdot x + 3x^2\cdot(-2) + 2\cdot x + 2(-2)$

$= \boxed{3x^3 - 6x^2 + 2x - 4}$

(21) $2x(2x^2+x-5)$ distribute 2x

$= 2x\cdot 2x^2 + 2x\cdot x + 2x\cdot(-5)$

$= \boxed{4x^3 + 2x^2 - 10x}$

(22) $-x^2(2x^3 - 3x + 4)$

$= -x^2\cdot(2x^3) + (-x^2)(-3x) + (-x^2)\cdot 4$

$= \boxed{-2x^5 + 3x^3 - 4x^2}$

(23) $(x+1)(3x^2 + x - 10)$

option 1: distribute

$x(3x^2 + x - 10) + 1(3x^2 + x - 10)$

$= 3x^3 + x^2 - 10x + 3x^2 + x - 10$

$= \boxed{3x^3 + 5x^2 - 9x - 10}$

option 2: arrows

$(x+1)(3x^2 + x - 10)$

$= x\cdot 3x^2 + x\cdot x + x\cdot(-10) + 1\cdot(3x^2) + 1\cdot x + 1(-10)$

$= 3x^3 + x^2 - 10x + 3x^2 + x - 10$

$= \boxed{3x^3 + 4x^2 - 9x - 10}$

option 3: boxes

	$3x^2$	x	-10
x	$3x^3$	x^2	$-10x$
1	$3x^2$	x	-10

$= \boxed{3x^3 + 4x^2 - 9x - 10}$

24 $3a^2b(2a^3-3ab+b^2)$ distribute $3a^2b$

$= 3a^2b\cdot 2a^2 + 3a^2b\cdot(-3ab) + 3a^2b\cdot b^2$

$= \boxed{6a^4b - 9a^3b^2 + 3a^2b^3}$

25 $3n(mn^3+5m)(m^2n-5n)$

option 1: FOIL first

$= 3n(mn^3\cdot m^2n + mn^3\cdot(-5n) + 5m\cdot m^2n + 5m(-5n))$

$= 3n(m^3n^4 - 5mn^4 + 5m^3n - 25mn)$

distribute next

$= 3n\cdot m^3n^4 + 3n(-5mn^4) + 3n(5m^3n) + 3n(-25mn)$

$= \boxed{3m^3n^5 - 15mn^5 + 15m^3n^2 - 75mn^2}$

option 2: distribute first

$(3n\cdot mn^3 + 3n\cdot 5m)(m^2n-5n)$

$= (3mn^4 + 15nm)(m^2n-5n)$

FOIL next

$= 3mn^4\cdot m^2n + 3mn^4(-5n) + 15nm\cdot m^2n + 15nm(-5n)$

$= \boxed{3m^3n^5 - 15mn^5 + 15m^3n^2 - 75mn^2}$

26 $(x+2)(x-2)$

$= x^2 - 2x + 2x - 4$

$= \boxed{x^2-4}$

This pattern is called
a <u>difference of two squares</u>

27 $(3-5x^2)(3+5x^2)$

$= 9 + 15x^2 - 15x^2 - 25x^4$

$= \boxed{9-25x^4}$

(28) $(x+2)(x+2)$

$= x^2+2x+2x+4$

$= \boxed{x^2+4x+4}$

This pattern is called a perfect square trinomial.

(29) $(3-5x^2)^2$

$= (3-5x^2)(3-5x^2)$

$= 9-15x^2-15x^2+25x^4$

$= \boxed{9-30x^2+25x^4}$

or

$= \boxed{25x^4-30x^2+9}$

(30) $(x-2)(x^2+2x+4)$

$$\begin{array}{rl} = & x^3+2x^2+4x \\ & \quad\ -2x^2-4x-8 \\ \hline = & \boxed{x^3-8} \end{array}$$

This pattern is called a difference of two cubes

(31) $(x+2)(x^2-2x+4)$

$$\begin{array}{rl} = & x^3-2x^2+4x \\ & \quad\ +2x^2-4x+8 \\ \hline = & \boxed{x^3+8} \end{array}$$

This pattern is called a sum of two cubes.

(32) $(x-2)^3$

$= (x-2)(x-2)(x-2)$

$= (x-2)(x^2-2x-2x+4)$

$= (x-2)(x^2-4x+4)$

$$\begin{array}{rl} = & x^3-4x^2+4x \\ & \quad\ -2x^2+8x-8 \\ \hline = & \boxed{x^3-6x^2+12x-8} \end{array}$$

This pattern is called a perfect cube.

33 $(x+2)^3$

$$= (x+2)\underbrace{(x+2)(x+2)}$$

$$= (x+2)(x^2+2x+2x+4)$$

$$= (x+2)(x^2+4x+4)$$

$$= x^3+4x^2+4x$$

$$+2x^2+8x+8$$

$$= \boxed{x^3+6x^2+12x+8}$$

another perfect cube.

(1) $2x^2yz^3(-7xy^3z^5)$ — parentheses means multiplication

$= 2(-7)\cdot x^2\cdot x\cdot y\cdot y^3\cdot z^3z^5$ — multiply like with like

$= \boxed{-14x^3y^4z^8}$ — add exponents

(2) $-6x^3y(3xyz - 9x^2z + 7y^2z^4)$

(1st ↑ 2nd ↑ 3rd)

inside parentheses, there are three terms, separated by add and subtract.

$= -6x^3y\cdot 3xyz - 6x^3y(-9x^2z) - 6x^3y(7y^2z^4)$ — distribute

$= \boxed{-18x^4y^2z + 54x^5yz - 42x^3y^3z^4}$ — multiply like with like, add exponents

(3) $(3x+7)^2$ — exponent outside parentheses.

$= (3x+7)(3x+7)$

CAUTION: Not: $3^2x^2 + 7^2$ ↑ no exponent law for add

exp law: $(xy)^2 = x^2y^2$ multiply

distribute 3x to both terms
then distribute 7 to both terms
also called FOIL: first-outside-inside-last

$= 3x\cdot 3x + 3x\cdot 7 + 7\cdot 3x + 7\cdot 7$

$= 9x^2 + 21x + 21x + 49$ — simplify

$= \boxed{9x^2 + 42x + 49}$ — combine like terms

(4) $(6x+5)(6x-5)$

$= 36x^2 - 30x + 30x - 25$ — The middle terms cancel out because one is (+) and the other (−).

$= \boxed{36x^2 - 25}$

This is a pattern: two terms separated by subtract is a difference.
Each term is a perfect square $(6x)^2 = 36x^2$
$5^2 = 25$
So this pattern is a difference of squares. $(a^2 - b^2)$

5 $(3x-2)(9x^2+6x+4)$

dist $3x$ to all 3 terms
dist -2 to all 3 terms

$= 27x^3 + 18x^2 + 12x - 18x^2 - 12x - 8$

can also be written vertically to line up like terms

$$\begin{array}{rrrr} = 27x^3 & +18x^2 & +12x & \\ & -18x^2 & -12x & -8 \\ \hline \end{array}$$

combine like terms

$= \boxed{27x^3 - 8}$

This is a pattern: two terms which are cubes $(3x)^3 = 27x^3$
$(2)^3 = 8$
separated by subtraction is called a difference of cubes $(a^3-b^3.)$

* Our goal when factoring is to remember how to construct $(3x-2)(9x^2+6x+4)$ when we see $27x^3-8$!

6 $(a+4b)(a^2-4ab+16b^2)$

same method as 5

$$\begin{array}{rrrr} = a^3 & -4a^2b & +16ab^2 & \\ & +4a^2b & -16ab^2 & +64b^3 \\ \hline \end{array}$$

$= \boxed{a^3 + 64b^3}$

This is a pattern: two terms which are cubes $(a)^3 = a^3$
$(4b)^3 = 64b^3$
separated by addition is called a Sum of cubes (x^3+y^3)

* When factoring, we will construct $(a+4b)(a^2-4ab+16b^2)$ when we see a^3+64b^3!

7 $(3x-2)^3$

Like problem 3, exponent is outside ( ), and because there's a subtraction inside the parentheses, no exponent law works.

$= \underbrace{(3x-2)(3x-2)(3x-2)}$

Because this is a power of 3, this is called a <u>perfect cube</u>.

<u>Step 1</u>: FOIL two of the factors.

$= (3x-2)(9x^2-6x-6x+4)$ combine like terms

$= (3x-2)(9x^2-12x+4)$

<u>Step 2</u>: dist $3x$ then dist -2

$= 27x^3 - 36x^2 + 12x$
$\quad\quad -18x^2 + 24x - 8$

<u>Unlike</u> the sum of cubes and difference of cubes patterns because middle terms do not cancel out!!

$= \boxed{27x^3 - 54x^2 + 36x - 8}$

A perfect cube is very difficult to factor. We leave that to Math 101/244.

8 Evaluate $f(x) = 6x^2 - x + 2$ when $x = a-1$

Replace x by $(a-1)$ using parentheses.

$f(a-1) = \underbrace{6(a-1)^2}_{\text{mult \& exponent}} - (a-1) + 2$

mult & exponent $\Rightarrow$ order of operations says exponent first

$(a-1)^2 \rightarrow (a-1)(a-1)$

$= 6(a-1)(a-1) - a + 1 + 2$ dist negative

$= 6(a^2-2a+1) - a + 3$ FOIL, combine like terms

$= 6a^2 - 12a + 6 - a + 3$ dist

$= \boxed{6a^2 - 13a + 9}$ combine

(9) Evaluate $f(x+h)$ when $f(x) = 6x^2 - x + 2$

replace x by $(x+h)$ using parentheses

$$\begin{aligned} f(x+h) &= 6(x+h)^2 - (x+h) + 2 && \text{as in (8), exponent} \\ &= 6(x+h)(x+h) - x - h + 2 && \text{dist neg} \\ &= 6(x^2 + 2xh + h^2) - x - h + 2 && \text{FOIL} \\ &= \boxed{6x^2 + 12xh + 6h^2 - x - h + 2} && \text{nothing combines!} \end{aligned}$$

(10) Evaluate $\dfrac{f(x+h) - f(x)}{h}$ when $f(x) = 6x^2 - x + 2$

* Notice * This $f(x)$ is the same as in previous problem!

$$\frac{\underbrace{f(x+h)}_{\text{replace by previous}} - \underbrace{f(x)}_{\text{replace by } f(x)\text{, using parentheses}}}{\underbrace{h}_{\text{just a letter, leave unchanged.}}}$$

$$= \frac{(6x^2 + 12xh + 6h^2 - x - h + 2) - (6x^2 - x + 2)}{h}$$

$$= \frac{6x^2 + 12xh + 6h^2 - x - h + 2 - 6x^2 + x - 2}{h}$$

subtract $\Rightarrow$ distribute negative

$$= \frac{12xh + 6h^2 - h}{h}$$

combine like terms

$6x^2 - 6x^2 = 0$

$-x + x = 0$

$2 - 2 = 0$

$$= \frac{12xh}{h} + \frac{6h^2}{h} - \frac{h}{h}$$

divide each term by h

$$= \boxed{12x + 6h - 1}$$